Ubiquitous drawing errors in the Simple Pendulum

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Abstract

The so-called simple pendulum is present in all introductory Physics textbooks, and one or more figures are usually devoted to its study. However, textbook authors seem to exercise little care in drawing the force diagrams for the motion of the pendulum. One finds mistakes in these drawings or features that may induce errors in the reader. In particular, a big confusion exists regarding the correct relative magnitude of the forces acting on the swinging mass.

The so-called simple pendulum provides a basic model which is present in all introductory Physics courses for students majoring in science and enginyeering. The dynamics of a pendulum is also a recurrent exercise in elementary dynamics, and all Physics textbooks devote one or more figures to this study. Nowadays, even major international projects address the pendulum.[1]

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Every year we pass a test to prospective Physics teachers about the forces that act on a simple pendulum oscillating between two points (A and A' in fig. 1), and we ask them to draw the force diagrams for each of the positions A, B and C as well as the resulting force and acceleration in each case; we also require students to draw these forces approximately to relative scale. The enormous failure rate in this simple exercise[2] has repeatedly shocked us. And one of the causes for students' errors is, we believe, the little care that textbooks authors exercise in drawing the force (and acceleration) diagrams for the motion of the pendulum.

Figures play an important role in facilitating students' understanding. From a good drawing, one may grasp the basic physical principles at play, although bad drawings may also contribute to generate misconceptions.[3,4] Images are still more abundant in our picture-dominated society and it is, therefore, surprising to find widespread and recurrent misguiding or even wrong drawings about the simple pendulum, in virtually all the widely used introductory Physics textbooks. The mistakes typically relate to the relative magnitude of the string tension and the normal component of the weight, as well as the direction of the resulting acceleration of the mass; these errors are not usually corrected in later editions of these textbooks.

The derivation of the angular dependence of the forces and the accelerations intervening in the oscillation appears in most textbooks and has been addressed in detail in this journal.[5] Measurements of accelerations and tensions have also been provided that help clarify the physics.[6,7] At any point in the trajectory of the swinging mass, the weight (W) and the string tension (T) are the only acting forces. One usually resolves the weight in a tangential (W_T) and a normal (W_N) component, and similarly the acceleration. The maximum tension always occurs at the bottom of the swing and the tension is minimum at the initial displacement (the position of maximum swing, $?_0$). These results are shown in fig. 2 for the points A, B and C indicated in fig. 1.

A case of a misguiding drawing in the treatment of the pendulum is one showing centripetal forces in the maximum position of swing. For example, although Resnick and Halliday's[8] clearly explain the need for a centripetal force in order to maintain the circular motion, and represent correctly the forces acting on the mass (fig. 15.10 in ref. 8), they represent them for the particular position of maximum elongation, which is the only position where no centripetal component of the acceleration exists because the mass is at rest at that position and, therefore, $T = W_N$. A similar diagram is used by Sears[9] (fig. 14.10 in ref. 9).

Roller and Blum[10] make T much larger than W_N at apparently the maximum elongation (fig. 16.8 in ref.10).

Another case of a wrong drawing is not showing clearly that $T > W_N$ at $? \neq ?_0$ (a condition for a centripetal force to exist). This appears in Alonso and Finn[11] (fig. 12.7 in ref. 11), where $W_N >> T$. In a more recent edition, Alonso and Finn[12] modify the drawing but the mistake is not completely corrected: W_N is still practically equal to the tension T at $? \neq ?_0$ (fig. 10.6 in ref. 12). The same mistake occurs in Gartenhaus' textbook[13] (fig. 6-12 in ref. 13). On the other hand, in a recent edition Halliday et al.[14] draw W_N larger than T (fig. 16.9.b in ref. 14).

Other mistakes refer to the resulting acceleration of the swinging mass and can be found in Sears[15] (fig. 11.8 in ref. 15) and in Gerthsen et *al*.[16] (fig. 1.2.5 in ref. 16). From their force diagrams one infers that the resulting force and, therefore, also the acceleration, should be tangent to the trajectory at all times; this, of course, is wrong because the total force and, therefore, the total acceleration constantly rises as the mass descends, and reaches the vertical position in the central position C, fig. 2. Gerthsen et *al.*[16] even draw a tangent acceleration at a position $? \neq ?_0$. Tipler,[17] in its 1977 edition, incorrectly makes W_N slightly larger than T also for $? < ?_0$. (fig. 14-9 in ref. 17). In Tipler's 1992 edition[18] (fig.12-13 in ref. 18) the drawing is modified by assigning, also incorrectly, the same magnitude to both quantities at an intermediate position in the trajectory. In a still further edition, Tipler[19] modifies the drawing again and makes it still more confusing to the student by drawing $W_N >> T$, also for an intermediate position (fig. 14.14 in ref. 19).

In conclusion, in the analysis of the pendulum in each of the textbooks reported here the force diagram or the acceleration of the swinging mass (either for the position of maximum elongation or for a general point in the trajectory) is either wrong or may induce errors in the reader. A big confusion exists in general regarding the correct relative magnitude of the acting forces. A more subtle misconception was highlighted by Schwarz[5] in that the analogy between the oscillatory motions of both a block on a spring and a swinging pendulum leads students to believe that the acceleration is zero at the bottom of the swing, where the velocity is maximum.

Overall, one is reminded of the following limerick by Wallach:[20]

If you are an author of books

You'd better beware of what cooks.

The figures you draw

You think have no flaw

Until viewed by a student who looks.

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Figure captions

Fig. 1: Three points, A, B and C, in the trajectory of a simple pendulum that oscillates between A and A'.

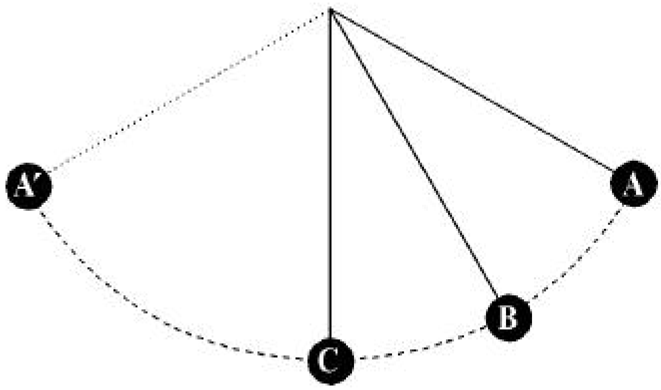

Fig. 2: Mass weight, string tension, normal and tangential components of the weight, and the resulting total force (and, therefore, the direction of the resulting acceleration) for points A, B and C in fig. 1.